

CHARACTERISING NEURONAL REPRESENTATION IN TERMS OF CELL MEMBRANE ACTIVITY: A APPROACH USING DIFFERENTIAL GEOMETRY.

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ABSTRACT

The receptive field of a neuron in primary visual cortex is classically defined as the region of visual space where the presence of a stimulus causes the neuron to increase its firing rate. In addition the neuron is said to be selective for a particular feature of the visual stimulus if it preferentially responds when that feature appears in its receptive field as opposed to other features. The view is also generally held that the neural representation, or code, of the feature is contained in the train of action potentials or spikes which the neuron produces in response, either as a “rate code” or as a “temporal code”. In this paper we put forward the hypothesis that the response of an individual neuron to the presence in a particular area of visual space of a particular component feature is not expressed solely in the sequence of action potentials it emits in response, but also by the *membrane potential* of the neuron, across the entire extent of its surface, including the axon hillock, performing some distinguished trajectory over space and time. Inspired by this hypothesis, which extends the notion of neural representation from *neural firing activity* to overall *membrane potential activity*, including *subthreshold activity*, we describe in the paper how a precise mathematical formulation of neural representation based on spatiotemporal membrane potential activity might be approached, using some concepts from differential geometry.

That the spatiotemporal pattern of subthreshold membrane potential activity is important in determining how a neuron represents a stimulus is given support by recent experimental results [1-3]. These indicate that the extent of the synaptic integration fields of neurons in the primary visual cortex spans a much larger area of visual space than that which is defined classically as the receptive field of the neuron as defined on the basis of spiking activity. In addition these experiments suggest that subthreshold responses to stimuli in a region surrounding the classical receptive field result from the integration of visual activation waves spread by slowly conducting horizontal axons intracortically. This therefore suggests that a representation of the visual scene by the overall membrane potential activity would contain not only the visual information that is currently being presented in the centre of the classical receptive field, but also the contextual information which was present in the surround a few tens of milliseconds earlier.

In order to develop a mathematical formulation of this form of neural representation, we regard the trajectory of the cell’s membrane potential as the solution to a system of first order nonlinear differential equations which describe the evolution of the membrane potential of a neuron over space and time. Thus the trajectory which a neuron’s membrane potential performs over any given period of time depends, as for all dynamical systems, on three factors. Firstly, it depends on the *initial conditions* or *initial state* of the neuron, ie the spatiotemporal distribution of membrane potential at the start of the period. Secondly, it depends on the *inputs* that the cell receives during the specific period. These inputs are the changes in membrane conductance which occur when activity of a presynaptic neuron causes the release of neurotransmitter into the synaptic cleft, and the uptake of this at the postsynaptic membrane causes the opening of specific ion channels and a corresponding inward or outward current flow across the cell membrane. The inputs to the cell have both a spatial and a temporal description, ie they appear at synapses on different points in the dendritic tree and at different times. Thirdly, it depends on the *intrinsic dynamic properties* of the cell. These are determined by such factors as the morphology, eg the branching structure, of the cell’s dendritic tree; the passive electrical properties of the cell membrane; the type and distribution of active conductances across the

extent of the cell membrane; and the state of the environment of the cell, eg the extracellular presence of particular chemicals at particular concentrations.

Given these three factors, the cell membrane potential will perform, over a given period of time, a certain spatial and temporal trajectory, which may or may not include the elicitation of so-called “spikes”. In particular, we can think of the inputs to the cell as *driving* the membrane potential along a particular trajectory in time and space, the trajectory being determined by the intrinsic dynamic properties of the cell.

The membrane potential is continually changing in this way, both externally driven by the spontaneous activation of its synapses, and as a result of autonomous, internal produced, variations in membrane conductance. Thus if the spatiotemporal activity of the neuron’s membrane potential is the neural representation for a particular feature in the stimulus, this representation must take the form of some kind of *distinguished* trajectory. For example, it has been observed that membrane potentials of visual cortical neurons in general exhibit large, low frequency fluctuations between a hyperpolarised and a depolarised state, presumably driven by alternating patterns of synaptic activity, and representing bistability in the surrounding cortical network [4]. Stimulation of the neuron results in a change in the dynamics of these fluctuations, reflected by an increase in the proportion of time that the membrane potential spends in the depolarised state.

In order to put these ideas into a mathematical framework, we consider in the paper a general form of a simplified neuron model, with n compartments and m synapses, described by the following first order system of differential equations:

$$\frac{dx}{dt} = f_0(t) + \sum_{i=1}^m u_i(t) f_i(x(t)) \quad x(0) = x_0 \in \mathbb{R}^n \quad (1)$$

In this equation, $x(t)$, the n -dimensional vector of membrane voltages, represents the state of the cell, and each $u_i(t)$ represents an input to the cell, in the form of a synaptic conductance. Note that we assume that the inputs appear in a multiplicative way in the differential equations, which is a reasonable assumption at least for AMPA- and GABA-based receptors at this simplified level of description of the cell dynamics. The functions $f_i(t)$ are defined by the intrinsic properties of the neuron, as determined by, for example, its dendritic morphology and the distribution and type of passive and active conductances throughout the extent of the cell’s membrane.

Based on the description given by (1), we show how the membrane potential can be seen as evolving on a *manifold*, ie some subset M of appropriately high dimension, of a correspondingly high-dimensional vector space. At each point in time t , the state of the neuron, expressed as a vector $x(t)$ of membrane potentials, measured at n discrete points on the cell membrane, lies at the corresponding point on the manifold M . Its movement from this point over the next infinitesimal period of time $(t, t+\Delta t)$ is determined by a *vector field* v defined on the manifold M . The effect of the inputs to the neuron will be to drive the trajectory of $x(t)$ to follow a particular curve, determined by the vector field v . The form of the vector field v is entirely determined by the intrinsic dynamic properties of the neuron.

The interesting question that we then address in the paper is how we might characterise the manifold M , and in particular the vector field on M which defines the membrane potential response elicited by a specific stimulus-related synaptic input, thereby characterising the neural representation of the neuron for the visual stimulus.

Keywords: neural representation, membrane potential, nonlinear dynamics

References

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