EFFECT OF A PERIODIC STIMULUS ON A NEURONAL DIFFUSION MODEL WITH SIGNAL DEPENDENT NOISE

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ABSTRACT

The Ornstein-Uhlenbeck neuronal model subject to periodic modulation is considered in the case where the amplitude of the noise is dependent on the incoming signal. Indeed it has recently been shown (cf. [2]) that under specific conditions the effect of considering signal-dependent noise in LIF models on the firing pattern can be substantial. It is then of interest to investigate on how resonant effects appearing when periodically time-varying signals are introduced in the model (cf. for example [1], [3]) are changed as a consequence of the signal dependency of the noise amplitude.

Several approaches can be followed to make the noise amplitude depend on the time-varying signal. Here we limit ourselves to consider the case where the global intensities of excitatory and inhibitory input processes, $\lambda_{mod} = \lambda + A\cos(\varpi t + \phi)$ and $\beta_{mod} = \beta + B\cos(\varpi t + \phi)$ respectively, are periodically modulated while the amplitudes a of PSPs are kept invariant in time. The resulting model X_t is then solution to the stochastic differential equation

$$\begin{cases} dX_t = \mu(x,t)\,dt + \sigma(t)dW_t \\ X_0 = 0 \end{cases} \tag{1}$$

with

$$\begin{aligned} \mu(x,t) &= -\frac{x}{\theta} + \mu + a(A-B)\cos(\varpi t + \phi); \\ \sigma^2(t) &= a^2\left(\lambda + \beta + (A+B)\cos(\varpi t + \phi)\right) \end{aligned} \tag{2}$$

where θ is the time constant, $\mu = a(\lambda - \beta)$, $\varpi = 2\pi/T$ with T the modulation period and ϕ its initial phase. The conditions $A < \lambda$ and $B < \beta$ must be fulfilled. We consider for simplicity the situation where the phase is reset to its initial value after each firing. In this way the series of interspike intervals generate a renewal process.

We choose two different sets of intrinsic and modulation parameters for model (1). In the first case, in accordance with [1], [3], we consider $\theta = 1/0.006$ ms^{-1}, fixed $\mu = 0.1$ mVms^{-1} and a value of the firing threshold $S = 20$ mV. For the second case we choose $\theta = 10$ ms^{-1}, fixed $\mu = 0.8$ mVms^{-1} and $S = 10$ mV, which are within generally accepted ranges for the intrinsic parameters (cf. [2]). The choice for the modulation parameters is made in a way that ensures to remain always in the subthreshold regime. Several criteria could be adopted to select the parameters λ, β and A, B. Here we fix a constant value for μ and for $a = 0.2$ mV, thus determining $\lambda - \beta$. Moreover, we consider the modulation amplitudes to be proportional on λ, β by a constant $k < 1$ and we assign to k different values.

We focus on the behavior of the ISI distribution for model (1) for a fixed modulation period T and initial phase $\phi = 0$ in each of the two cases mentioned above. In particular, we consider the phase-locking phenomenon and the dependence of the height of the peak at T for the ISI distribution on the noise intensity, which is associated for each set of parameters with the value $\sigma^2(T)$. A comparison is performed with the corresponding results obtained for processes with noise not dependent on the input.

Our study allows to pinpoint the following features characterising the cases with input-dependent noise with respect to the case without the dependence:
a. the phase-locking behavior is more precise in the sense that the ISI distributions show deeper minima;
b. for increasing values of $\sigma^2(T)$ the range where the phase-locking behavior, i.e. the multimodal shape of the ISI distribution with peaks located near integer multiples of the modulation period, is maintained is wider; moreover, such range increases with increasing values of the modulation amplitudes A and B;
c. the heights of the ISI distribution at T are always higher than for the corresponding processes with noise not dependent on the input.

Keywords: Noisy integrate-fire model, periodic stimulus, ISI distribution.

References

[1] Bulsara, A.R., Elston, T.C., Doering, C.R., Lowen, S.B. and Lindenberg, K. (1996) Cooperative behavior in periodically driven noisy integrate-fire models of neuronal dynamics. *Phys. Rev E* **53, 4**: 3958-3969.

[2] Lánský, P. and Sacerdote, L. (2001) The Ornstein-Uhlenbeck neuronal model with signal-dependent noise. *Phys. Lett. A* **285**: 132-140.

[3] Shimokawa, T., Pakdaman, K. and Sato, S. (1999) Time-scale matching in the response of a leaky integrate-and-fire neuron model to periodic stimulus with additive noise. *Phys. Rev. E* **59**: 3427-3443.